Pergamon

Int. J. Multiphase Flow Vol. 23, No. 3, pp. 493–501, 1997

Printed in Great Britain
0301-9322/97 $17.00 + 0.00

PII: S0301-9322(96)00084-5

SIMPLIFIED TRANSIENT SIMULATION OF TWO PHASE FLOW USING QUASI-EQUILIBRIUM MOMENTUM BALANCES

Y. TAITEL and D. BARNEA

Department of Fluid Mechanics and Heat Transfer, Faculty of Engineering, Tel-Aviv University, Ramat-Aviv 69978, Israel

(Received 5 September 1995; in revised form 5 November 1996)

Abstract—Transient simulations of two phase gas-liquid flow in pipes require considerable computational efforts. Available commercial Codes that were developed, or are being developed at the present, use either the two fluid model or the drift flux model. These models implement continuity and momentum transient equations.

In many occasions, in particular for the oil and gas industry, the transient response is usually relatively slow. Thus, it is suggested here to use quasi equilibrium momentum equations for the liquid and the gas together with transient continuity equations. This procedure results in a simplified numerical algorithm which can be used as an efficient transient simulator.

Key Words: two phase flour, transient simulation

1. INTRODUCTION

Transient simulations of gas-liquid flow in pipelines require major computing efforts leading usually to elaborate computer Codes. Recently several Codes were developed, supported by the oil industry. The first and the most well known Code is 'OLGA' that was developed in Norway (Bendiksen *et al.* 1987, 1991). Later the Code 'PLAC' (Black *et al.* 1990) was developed in England, 'TACITE' is now being developed in France (Fabre *et al.* 1989; Pauchon *et al.* 1993, 1994). Shell Oil Company developed 'TRAFLOW'. All Codes are based on the two fluid model or the drift flux model in which transient formulation for the continuity and the momentum equations is used.

The development of two-phase transient simulators was initiated by the nuclear industry which developed the basic strategy of the solution. In the nuclear industry fast transients are of major interest while in the oil and gas industry the interest is usually in relatively slow transients. Under these conditions one may consider the momentum equations to be in quasi-equilibrium leading to the use of simpler and less elaborate calculations.

Taitel *et al.* (1989) proposed a simplified model in which the only transient equation was the liquid continuity equation. The gas continuity equation and the momentum equations for the liquid and the gas were assumed in a quasi-steady state. The approach seems to yield good results under conditions of fast gas flow rates (Minami 1991; Minami and Shoham 1994). But, it lacks the ability to treat correctly the gas accumulation in the pipe.

A recent review on the available numerical Codes that include a comparison of the performance of OLGA, PLAC and the aforementioned simplified method (Taitel *et al.* 1989) was reported by Vigneron *et al.* (1995).

In this work a simplified numerical method that treats correctly the continuity equations of both the gas and the liquid is proposed. The momentum equations, however, for the liquid and the gas, are assumed to be in local quasi-equilibrium. The end result is a simplified numerical scheme which can easily implement steady state models for flow pattern transition and flow rate calculations.

2. ANALYSIS

The transient model proposed here is a simplified approach which can be used adequately for the prediction of the transient behavior of two-phase, gas–liquid flow, in pipes. The formulation is fully transient with respect to the continuity equations but it assumes local quasi equilibrium for the momentum equations for the liquid and the gas, that is, the existence of a force balance in each section of the pipe. This procedure takes advantage of the fact that the gas is compressible and its density is a function of the pressure whereas the liquid is considered incompressible or only slightly compressible. The pipe is subdivided into N sections of length Δx (see figure 1). The numerical calculations consider the liquid and the gas holdups and the pressure in each section to be known at time t (including the initial time) and proceed to calculate the flow variables at time $t + \Delta t$, The calculation of the next time step of the holdup and the pressure is done in two stages. In the first stage the liquid holdup is kept constant and the flow rates of the gas and the liquid in each section along with the flow pattern are calculated, based on the pressure and void fraction. Then the new values of the gas mass, the gas density and the pressure in each section are calculated at time $t + \Delta t$. This phase of the calculation is performed by an implicit scheme which guarantees stability of the numerical procedure also for large time increments Δt. In the next stage the new liquid flow rate and the new liquid holdup are calculated, on the basis of the new pressure distribution. In this step a simple explicit procedure is applied. Note that abrupt variation with time can take place for the pressure and the pressure drop, owing to a change of the flow pattern. The holdup, however, varies only slowly with time as liquid has to move in or out to change the holdup in a section and this process takes time.

A key element of the calculation is the correct modeling of the flow hydrodynamics which allows the calculation of the flow rates once the pressure and holdup variations along the pipe are known. The development of such relations for separated flow and dispersed flow follows.

2.1. *Separated flow*

In separated flow the liquid and the gas are considered to flow side by side. This flow pattern includes the stratified flow case where the liquid is at the bottom of the pipe and the annular flow case where the liquid is spread around the pipe periphery. Assuming a quasi-equilibrium force balance on each section i yields (the subscript i is omitted for most variables),

$$(P_i - P_{i+1})A_G - \tau_G S_G \Delta x - \tau_I S_I \Delta x - \rho_G g A_G (H_{i+1} - H_i) = 0 \qquad [1]$$

for the gas and

$$(P_i - P_{i+1})A_L - \tau_L S_L \Delta x + \tau_I S_I \Delta x - \rho_L g A_L (H_{i+1} - H_i) = 0 \qquad [2]$$

for the liquid.

P is pressure, A is the cross sectional area, H is the elevation level of section i and ρ is the density. τ is the shear stress and S is the perimeter over which τ acts. L and G denote liquid and gas, respectively, and the subscript I denotes interface. The shear stresses can be correlated as follows,

$$\tau_G = f_G \frac{\rho_G U_G^2}{2}; \quad \tau_L = f_L \frac{\rho_L U_L |U_L|}{2}; \quad \tau_I = f_I \frac{\rho_G (U_G - U_L)|U_G - U_L|}{2}, \qquad [3]$$

where U is the axial average velocity and f is the friction factor. In stratified flow the gas is assumed to flow in the positive direction only (see [3]). The liquid, on the other hand can flow forward and backward.

For the shear stresses between the liquid or the gas and the pipe surface, the friction factors, f_L and f_G can is approximated by the correlation $f = CR_e^{-n}$ where $C = 0.046$, $n = 0.2$ for turbulent flow, and $C = 16$, $n = 1$ for laminar flow. The Reynolds numbers were defined as $R_{e_L} = 4U_L A_L \rho_L / S_L \mu_L$ for the liquid and $R_{e_G} = 4U_G A_G \rho_G / (S_G + S_I)\mu_G$ for the gas. For the interfacial gas–liquid shear stress we adopted the relation $f_I = 0.005(1 + 75A_L/A)$ (Wallis 1969). When f_I is larger than 0.05 then this value is used.

Equations [1] and [2] are used to calculate the mass flow rate (and the velocity) of the gas and the liquid in each section for given void fraction and pressure difference $P_i - P_{i+1}$. Equations [1] and [2] are non linear equations for U_G and U_L, thus some iteration is required to obtain the gas and the liquid flow rates. Note, however, that a good 'first guess' can be easily obtained for the case of constant friction factors and negligible liquid velocity ($U_G \gg U_L$), for which case simple quadratic equations for U_G and U_L are obtained. This calculation is performed at the beginning of each time step, for given pressure variation and liquid holdup. For simplicity annular flow is treated here the same way as stratified flow.

2.2. Dispersed flow

Slug flow and dispersed bubble flow, are treated here using the drift flux model. For this case the pressure drop is calculated by

$$(P_i - P_{i+1})A = \frac{1}{2} f_M \rho_M U_M^2 \pi D \Delta x + A \rho_M g(H_{i+1} - H_i) \quad [4]$$

where U_M is the mixture velocity ($U_M = U_{LS} + U_{GS}$). ϵ_G is the void fraction, ϵ_L is the liquid holdup and ρ_M is the mixture density ($\rho_M = \epsilon_G \rho_G + \epsilon_L \rho_L$). The subscript S indicate superficial velocity.

The gas velocity is correlated in the form

$$U_G = CU_M + U_d. \quad [5]$$

Substituting [5] in [4] yields a quadratic equation for the gas velocity U_G and the gas mass flow rate $Q_G = U_G \rho_G A_G$. Once U_G is known, U_L and Q_L can be calculated from [5] noting that $U_M = \epsilon_G U_G + \epsilon_L U_L$.

2.3. Solution algorithm

Based on these ideas the solution procedure can be described as follows:

(1) The procedure starts at time t where the liquid holdup ϵ_L, the pressure P and the flow pattern in each section of the pipe (see figure 1) are given. The inlet and outlet pressures, P_1 and P_N, and the liquid holdup at the entrance (the first section), ϵ_{L1}, are given as boundary conditions.

(2) The flow rates of the gas Q_{Gi} and of the liquid Q_{Li} at each section i are calculated using [1] and [2] for separated flow and [4] and [5] for dispersed flow. This calculation is based on the force balance in each section (a quasi equilibrium momentum balance) and it depends on the flow pattern. Once the flow rates are calculated the flow pattern is computed (will be described later) to make sure that the flow pattern used is correct. If not the calculation of Q_{Gi} and Q_{Li} is repeated with the new flow pattern.

(3) In this step, an implicit solution of the pressure variation along the pipe for the next time interval is carried out. The liquid level is kept constant and only the gas is allowed to flow from section to section during this step of the calculation.

For an efficient implicit solution of the pressure a linear relation between the gas flow rate at time $K+1$ and the pressure difference at this time ($K+1$) is needed, namely a relation of the form,

$$Q_{Gi}^{K+1} = W_i^K [P_i^{K+1} - P_{i+1}^{K+1}] + R_i^K. \quad [6]$$

Note that [1]–[3] and [4]–[5] which are used to calculate the flow rates for given pressure distribution and holdup do not yield a linear relation between the flow rate and the pressure difference.

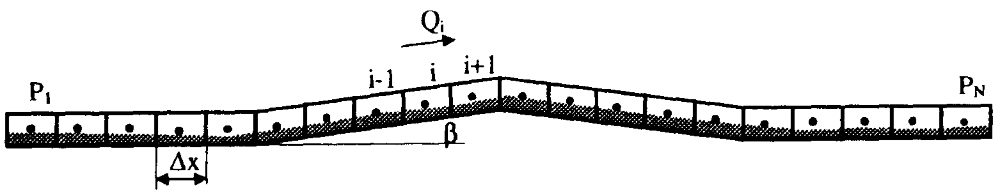

Figure 1. Geometry of the pipeline.

For the case of stratified flow equation [1] can be recast in the format of [6] if W_i^K and R_i^K take the form

$$W_i^K = \frac{1}{\left[\frac{(f_G S_G + \delta f_I S_I)\Delta x Q_G}{2A_G^3 \rho_G} - \frac{f_I S_I \delta \Delta x Q_L}{A_G^2 \rho_L A_L}\right]} \quad [7]$$

$$R_i^K = \frac{\rho_G g(H_i - H_{i+1}) - \frac{f_I S_I \rho_G \delta \Delta x Q_L^2}{2A_G \rho_L^2 A_L^2}}{\left[\frac{(f_G S_G + \delta f_I S_I)\Delta x Q_G}{2A_G^3 \rho_G} - \frac{f_I S_I \delta \Delta x Q_L}{A_G^2 \rho_L A_L}\right]}, \quad [8]$$

where $\delta = 1$ for $U_G > U_L$ and -1 for $U_G < U_L$. Note that all the variables in [7] and [8] are calculated at time K and thus are considered known.

For the case of dispersed flow [4] takes a linear format if:

$$W_i^K = \frac{2AC^2 \rho_G A_G}{f_M \rho_M \pi D \Delta x |U_G - U_d|} \quad [9]$$

and

$$R_i^K = -\frac{2AC^2 \rho_G A_G g(H_{i+1} - H_i)}{f_M \pi D \Delta x |U_G - U_d|} + \rho_G A_G U_d. \quad [10]$$

As mentioned, all the variables in [9] and [10] are also calculated at time K. The coefficients C and the drift velocity U_d depend on the flow pattern (slug flow or dispersed bubble flow) and on the inclination angle. In this work we used $C = 1.2$ and $U_d = 0.25$ m/s for slug flow and $C = 1$ and $U_d = 0$ for dispersed bubble flow. The implicit solution of the pressure distribution at time $t + \Delta t$ involves the following relations:

(a) The new gas flow rate is represented by the linear relation [6].
(b) The mass of the gas at the new time step $K + 1$ is calculated by

$$m_{Gi}^{K+1} = m_{Gi}^K + \Delta t(Q_{G,i-1}^{K+1} - Q_{G,i}^{K+1}). \quad [11]$$

(c) The new gas density is calculated by

$$\rho_{Gi}^{K+1} = \frac{m_{Gi}^{K+1}}{A_G^K \Delta x}. \quad [12]$$

(d) The new pressure P_i^{K+1} is calculated using the state equation. In this work we used the ideal gas equation of state, that is

$$P_i^{K+1} = \rho_{Gi}^{K+1} RT. \quad [13]$$

(e) Substituting [11] and [12] in [13] yields

$$P_i^{K+1} = y_i^K + z_i^K(Q_{G,i-1}^{K+1} - Q_{G,i}^{K+1}), \quad [14]$$

where

$$y_i^K = \frac{m_{G,i}^K RT}{A_{Gi}^K \Delta x}$$

and

$$z_i^K = \frac{RT\Delta t}{A_{Gi}^K \Delta x}.$$

(f) Substituting [6] in [14] yields a set of equations:

$$-z_i^K w_{i-1}^K P_{i-1}^{K+1} + (1 + z_i^K w_{i-1}^K + z_i^K w_i^K) P_i^{K+1} - z_i^K w_i^K P_{i+1}^{K+1} = y_i^K + z_i^K R_{i-1}^K - z_i^K R_i^K. \quad [15]$$

Equation [15] is an implicit set of equations for the pressure variation at the new time step $K+1$ for $i = 2 \ldots N-1$. Note that P_1 and P_N are prescribed as boundary conditions. This set of equations can be easily solved using the Thomas Algorithm.

(4) Based on the pressure profile at the new time step, the new gas flow rates are calculated using [6].
(5) The new gas density is calculated using the state equation $\rho_{Gi}^{K+1} = P_i^{K+1}/RT$.
(6) The new liquid density is calculated. In this work it is assumed that the liquid density is constant.
(7) The new gas mass is calculated using [11].
(8) The new liquid mass flow rate Q_{Li}^{K+1} is calculated based on the specific flow pattern at section i. Q_{Li}^{K+1} depends on P_i^{K+1}, Q_{Gi}^{K+1}, ρ_{Gi}^{K+1}, ρ_{Li}^{K+1}, etc. This dependence is calculated as in step 2.
(9) The new liquid mass in each section is

$$m_{Li}^{K+1} = m_{Li}^K + \Delta t(Q_{Lin}^{K+1} - Q_{Lout}^{K+1}). \quad [16]$$

Unlike the gas that is assumed to flow only in the downstream direction the liquid can flow 'backward' when the pipe inclination angle is uphill. Consider a pipe element i adjacent to the elements $i-1$ (on the left) and $i+1$ (on the right). The liquid flow rate in each element can be positive (in the downflow direction) or negative. Q_{Li} (in absolute value) is always consider as Q_{Lout}. The Q_{Lin} is taken as the liquid flow rate of the neighboring elements with the velocity towards section i. Note, for example, that for the 'normal' case when the flow is positive then $Q_{Lin} - Q_{Lout}$ is simply $Q_{Li-1} - Q_{Li}$.

(10) The new liquid holdup is calculated

$$\epsilon_{Li}^{K+1} = \frac{m_{Li}^{K+1}}{\rho_{Li}^{K+1} A \Delta x}. \quad [17]$$

(11) Change 'new' variables into 'old' and go back to step 1.

2.4. *Flow pattern*

The most serious problem with the proposed method is the determination of the flow pattern. For the case of steady flow, once the physical properties of the phases, the pipe diameter and the inclination angle are known the flow pattern is determined uniquely by the gas and the liquid flow rates. In the proposed simulation the flow rates are determined by the pressure and void distribution. One may find out that when stratified flow is assumed (for example) than the calculated pattern, which is based on the flow rates that were evaluated by the stratified assumption, will show transition to slug flow, and while in slug flow the flow rates obtained for the same pressure and void distributions will show transition to stratified flow. This problem could be solved in two ways. One way is to have a buffer zone for the transition criterion, that is, the transition from slug to annular (for example) will take place at a different location than the transition from annular to slug flow. Another way to solve this problem is to decide on a preferred flow pattern, that is, when we have oscillation in the flow pattern for the same holdup and pressure profile we should decide on a preferred flow pattern. In other words, when the solution for the flow pattern (and flow rates) is not unique we need to determine the flow pattern that will take place based on physical grounds. Both methods are utilized in this work.

In addition one has to take into account that the calculation of the transition boundaries should be relatively simple and quick so that the numerical solution is efficient and fast.

In this work the following criteria for transition were used:

The criterion used for the transition from separated flow to dispersed flow is a combination of the Kelvin–Helmholtz transition criterion and a high liquid level, as proposed by Taitel and Dukler (1976). Thus, transition to dispersed flow (slug and bubbly flow) will take place when,

$$U_G > \left(1 - \frac{h_L}{D}\right)\sqrt{\frac{(\rho_L - \rho_G)g \cos \beta A_G}{\rho_G S_I}} + U_L \qquad [18]$$

and $h_L/D > 0.35$, where h_L is the liquid level.

The transition from dispersed flow to separated flow is handled differently for the case of upward inclination and the case for horizontal and downward inclination.

For upward inclination we considered the transition to separated flow on the basis of the liquid holdup only. In this case a buffer zone is used, thus transition to separated flow will occur once the liquid holdup, ϵ_L, is less than 0.2 (equivalent $h_L/D < 0.25$) while the transition from annular flow to slug flow occurs at $h_L/D = 0.35$ (provided the interface is unstable).

For horizontal and downward inclinations the Kelvin–Helmholtz criterion is used to determine the transition from slug to separated flow. This is done in the following way: For given liquid and gas flow rates an hypothetical quasi-equilibrium liquid level is calculated. On the basis of the flow rates and the calculated liquid level the K–H criterion, [18] is used. If stable to K–H then the resulting flow pattern will be stratified flow, if unstable and the liquid holdup is less than 0.2, annular 'flow' will result in. Otherwise the flow will remain in slug or dispersed bubble flow.

As mentioned, annular flow is treated as stratified flow [1]–[3] and dispersed bubble flow is treated as slug flow [4], [5]. The 'internal' distinction between stratified flow and annular flow is based on the Kelvin–Helmholtz criterion and holdup. Thus 'separated' flow with $\epsilon_L < 0.2$ which is unstable to Kelvin–Helmholtz [18] is annular flow. Likewise the distinction in dispersed flow between slug flow and dispersed bubble flow is based of the liquid velocity. When the liquid velocity is higher than a critical velocity as described by Barnea (1986) dispersed bubble flow will result in, otherwise the flow will be in slug flow.

As mentioned, when the solution for the flow pattern is not unique and the flow patterns oscillates (step 2 in the suggested algorithm) one has to make a decision as to the preferred flow pattern that will physically exist. If for the same void fraction and pressure distribution one can obtain a stable and an unstable condition the unstable pattern is the dominant one. Thus if one obtains oscillations between stratified and slug flow the preferred pattern is the slug flow pattern.

It should be noted that the proposed criteria here are not a final recommendation for handling transient flow pattern transition. It is just a reasonable suggestion. Obviously, the numerical algorithm proposed here can accommodate any other transient flow pattern criteria for transition.

3. RESULTS AND DISCUSSION

The main purpose of this article is to present the ideas and the framework for a simplified numerical algorithm which is based on quasi-equilibrium momentum equations and transient continuity equations. Many variations can be implemented here and the use of the particular models for the calculation of the flow rates and the transition criteria are open to different approaches.

Nevertheless, in order to demonstrate the applicability of the method some examples of typical cases are presented in figures 2–4. In these figures the results for the liquid holdup and the pressure distributions are plotted for few selected times.

In figure 2 a horizontal pipe is considered, 800 m long and 5 cm diameter. The initial liquid holdup, ϵ_L; is 0.2, the outlet pressure is 2 atm and the inlet pressure is 2.2 atm. The initial pressure distribution is assumed to be linear. The liquid holdup at the entrance increases linearly with time from 0.2 to 0.5 in 300 s. The pipe is subdivided into 40 sections and the time increment chosen is 1 s. Figure 2 shows how the liquid holdup and the pressure vary with time until a steady state is reached. As can be seen the liquid penetrates into the pipe to fill half of the pipe. The pressure is initially linear, but, once the liquid penetrates the pipe, the pressure drop at the entrance region of the high holdup is larger than in the lower holdup region downstream. This can be clearly seen by the pressure distribution for $t = 300$, 500 and 700 s. At the end, however, when a final steady state is reached, the pressure variation is again linear. Because the pressure difference in this case

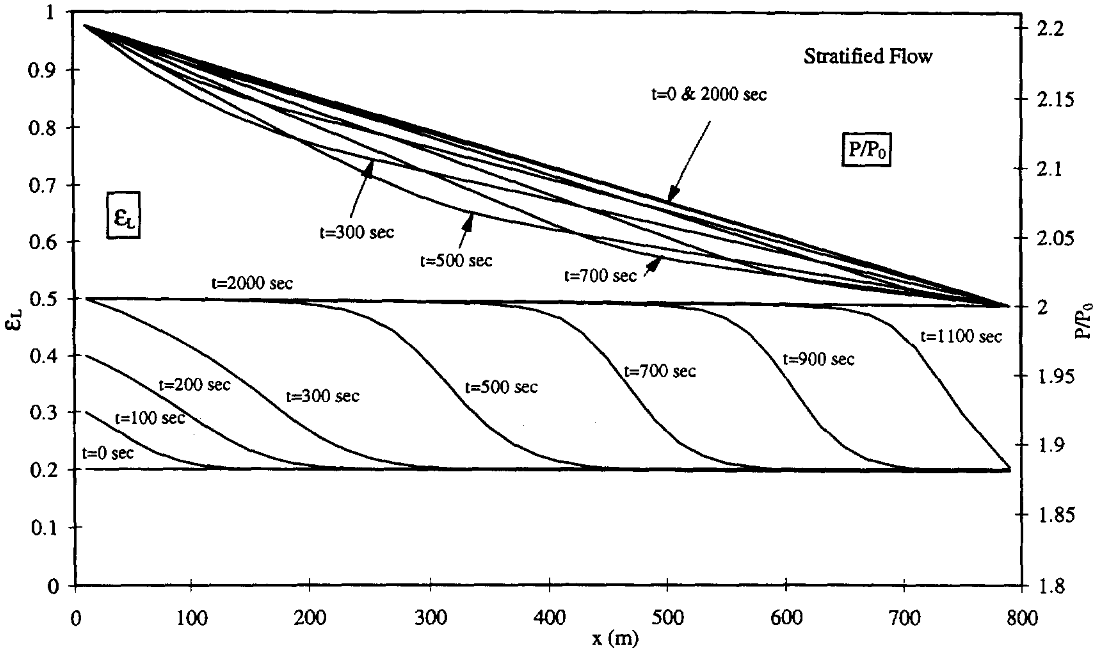

Figure 2. Transient profiles of liquid holdup and pressure. Horizontal pipe, increase inlet holdup, $P_1 = 2.2$ atm.

is low $\Delta P = 0.2$ atm the density of the gas is almost constant and in this case the pressure variation is almost linear. Also note that since the pressure difference is low, the liquid and gas velocities remain sufficiently low and the flow pattern remains stratified since instability due to Kelvin–Helmholtz is not reached in this process.

Figure 3 is a similar case with the only exception that the inlet pressure is 5 atm. In this case the flow patterns along the pipe change with time. Since the pressure difference this time is high,

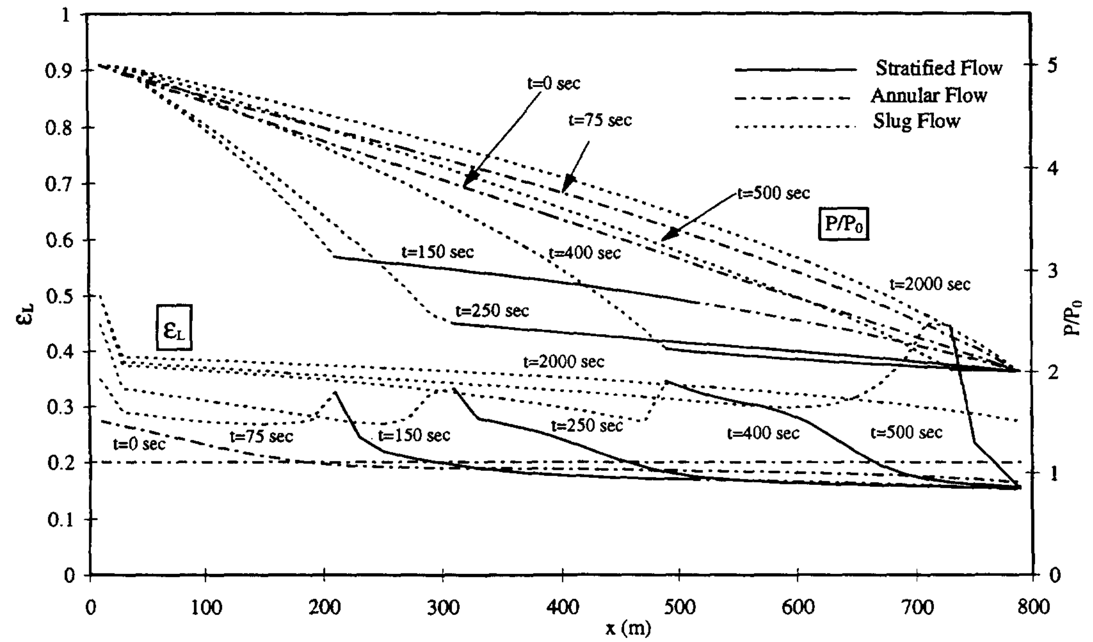

Figure 3. Transient profiles of liquid holdup and pressure. Horizontal pipe, increase inlet holdup, $P_1 = 5$ atm.

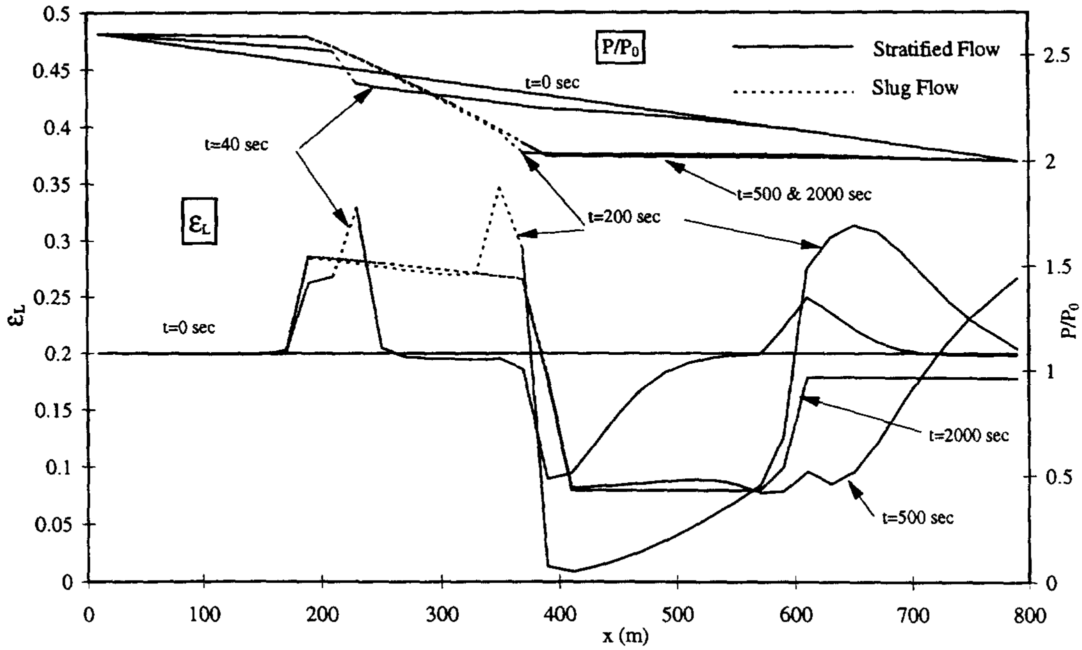

Figure 4. Transient profiles of liquid holdup and pressure. Inclined pipe, $P_1 = 2.6$ atm.

the liquid and gas flow velocities are sufficiently high to initiate the K–H instability and therefore, right at time $t = 0$, the flow pattern is annular. Once the liquid level rises near the entrance, transition to slug flow occurs, and a lump of liquid is being pushed, creating a hump in the liquid holdup that is being continuously pushed to the pipe exit. The case of $t = 150$ s is interesting, as one can observe that near the entrance the flow pattern is slug flow, after the hump it is stratified flow and near the exit it is annular flow. Note that the pressure drop for the slug region is much larger than the pressure drop for stratified flow. As the slugs are created, liquid and gas velocities decrease and the flow downstream which was in annular flow becomes stratified flow. Finally, a steady state is reached with the whole pipe in slug flow. Note that unlike the previous case, the final pressure drop is not linear, this is due to the gas expansion in the downstream direction resulting in an increase of the gas velocity and the pressure drop near the pipe exit. Also note the discontinuity in the holdup at the pipe entrance. This is due to the fact that at the entrance stratified flow is imposed. Once the flow pattern near the entrance is not stratified a jump to the correct holdup is shown in the next nearest section.

The last example is shown in figure 4. The 800 m pipe is subdivided into four parts, each 200 m long. The first pipe is horizontal, the second is 2° inclined upwards, the third is 2° inclined downwards and the forth is horizontal again (as shown in figure 1). Initially, the pipe is uniformly filled with liquid at a level of $h_L/D = 0.25$ ($\epsilon_L = 0.2$) and the initial pressure profile is assumed to be linear. Outlet pressure is taken as 2 atm and inlet pressure as 2.6 atm, both constant with time.

The initial conditions are not steady state conditions and the simulation shows the transient behavior from the present conditions until steady state is reached.

The initial flow pattern is of stratified flow. At the beginning the liquid is depleted at the top ($x = 400$ m) and is accumulated at the bends, $x = 200$ and 600 m. Note that the liquid flows backwards in the uphill sections. At time $t = 40$ s the accumulation of the liquid at the elbow ($x = 200$) results in transition to slug flow and increase of the pressure drop. At $t = 200$ s the flow pattern in the uphill sections is mostly slug flow. The high liquid holdup near the top results from the scooping of the liquid in the pipe, once transition to slug flow takes place. One can also observe at this time that the downward pipe is almost depleted of liquid which is accumulated in the horizontal end pipe. At time $t = 500$ s the upstream part is almost in steady state while the liquid is still being pushed to the outlet leaving liquid holdup at the region of the horizontal section low near the elbow ($x = 600$ m) and high near the exit. Finally, a steady state is reached as shown at $t = 2000$ s. In this case the flow pattern is stratified except in the uphill section. The pressure distribution shows that the pressure gradient is very low in the down inclination section and in the horizontal sections and is high in the uphill section. Note that the liquid holdup at the inlet horizontal section is higher than at the exit horizontal section. This is because of the decrease of the pressure downstream and the increase of the gas velocity. Likewise the holdup in the uphill section, which is in slug flow, decreases with distance for the same reason.

4. SUMMARY AND CONCLUSIONS

A numerical algorithm that uses two transient continuity equations for the liquid and the gas and quasi equilibrium momentum equations is proposed in this work.

The applicability of this approach is demonstrated by few examples that show typical transient behavior of two phase flow in a pipeline.

One of the main objective of the present work has been the development of a relatively simple and user friendly, yet efficient algorithm for the simulation of the time dependent dynamics of two phase flow in pipes. This algorithm can serve as a basis for further development and more complex applications.

REFERENCES

Barnea, D. (1986) Transition from annular flow and from dispersed-bubble flow—unified models for the whole range of pipe inclinations. *Int. J. Multiphase Flow* **12,** 733–744.

Bendiksen, K., Brandt, I., Jacobsen, K. A. and Pauchon, C. (1987) Dynamic simulation of multiphase transportation systems. Presented at the *Multiphase Technology and Consequences for Field Development Forum*, Stavanger, Norway.

Bendiksen, K., Malnes, D., Moe, R. and Nuland, S. (1991) The dynamic two-fluid model OLGA: theory and application. *SPE Production Engineering* **6,** 171–180.

Black, P. S., Daniels, L. C., Hoyle, N. C. and Jepson, W. P. (1990) Studying transient multiphase flow using the pipeline analysis Code (PLAC). *Journal of Energy Resources Technology* **112,** 25–29.

Fabre, J., Liné, A. and Peresson, L. (1989) Two fluid/two flow pattern model for transient gas-liquid flow in pipes. Presented at the *4th BHRA Multiphase Flow International Conference*, Nice, France.

Minami, K. (1991) Transient flow and pigging dynamics in two-phase pipelines. Ph.D. dissertation, University of Tulsa.

Minami, K. and Shoham, O. (1994) Transient two-phase flow behavior in pipelines—experiment and modeling. *Int. J. Multiphase Flow* **4,** 739–752.

Pauchon, C., Dhulesia. H., Lopes, D. and Fabre, J. (1993) TACITE: A comprehensive mechanistic model for two-phase flow. Presented at the *6th International Conference on Multiphase Production*, Cannes, France.

Pauchon, C., Dhulesia. H., Binh Cirlot, G. and Fabre, J. (1994) TACITE: A transient tool for multiphase pipeline and well simulation. SPE 28545, Presented at *1994 SPE Annual Technical Conference*, New Orleans, LA, U.S.A., pp. 25–28.

Taitel, Y., Shoham, O. and Brill, J. P. (1989) Simplified transient solution and simulation of two phase flow in pipelines. *Chem. Eng. Sci.* **44,** 1353–1359.

Vigneron. F., Sarica, C. and Brill, J. P. (1995) Experimental analysis of imposed two-phase flow transients in horizontal pipelines. Proceedings of the *BHR Group 7th International Conference, Multiphase 95*, Wilson, A., ed., pp. 199–217.

Wallis, G. B. (1969) *One Dimensional Two-phase Flow*. McGraw-Hill, New York.